# Reporting regression results

# Extracting model estimates and data structure

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# **Learning objectives**

Today we will...

- run our first linear (mixed) model
- produce model summary tables
- report analyses and cite packages
- report model results reproducibly

# Resources

# Set-up

# **Packages**

• we'll need the following packages for today

```
pacman::p_load(
  tidyverse,
  lme4,
  broom,
  broom.mixed,
  brms
)
```

#### Data

• we'll need the dataset we've been working with

### **Data dictionary**

• let's review the data dictionary

```
dict_lifetime <- read_csv(
  here::here("data", "tidy_data_lifetime_pilot_dictionary.csv")
)</pre>
```

### Model prep

• we want to only have critical items

```
df_lifetime <-
  df_lifetime |>
  filter(type == "critical") |>
  droplevels()
```

• we want our predictors (lifetime and tense) to be factors

```
df_lifetime$lifetime <- as.factor(df_lifetime$lifetime)
df_lifetime$tense <- as.factor(df_lifetime$tense)</pre>
```

```
• and to have sum contrast coding:
```

```
- PP and living are -0,5
```

- SF and dead are +0,5

#### contrasts(df\_lifetime\$lifetime)

```
\begin{array}{cc} & \text{living} \\ \text{dead} & 0 \\ \text{living} & 1 \\ \end{array}
```

### contrasts(df\_lifetime\$tense)

```
SF
PP 0
SF 1
```

- we see the default treatment (or 'dummy') contrasts here
- to change them:

```
contrasts(df_lifetime$lifetime) <- c(-0.5, +0.5)
contrasts(df_lifetime$tense) <- c(+0.5, -0.5)
```

### contrasts(df\_lifetime\$lifetime)

```
[,1] dead -0.5 living 0.5
```

### contrasts(df\_lifetime\$tense)

```
[,1]
PP 0.5
SF -0.5
```

# 1 Linear regression

- fitting a straight line to data
- for an introduction see webbook for my course Regression for Linguists, e.g., Ch. 1 Understanding straight lines
- fixed-effects only multiple regression:

```
lm(
  log(fp) ~ tense * lifetime,
  subset = region == "verb" & ff > 0,
  data = df_lifetime
)
Call:
lm(formula = log(fp) ~ tense * lifetime, data = df_lifetime,
    subset = region == "verb" & ff > 0)
Coefficients:
     (Intercept)
                                            lifetime1 tense1:lifetime1
                            tense1
         5.63477
                                                               -0.08320
                           0.03357
                                            -0.10130
```

### 1.1 Naming conventions

- it's very helpful to save our models to our environment so we can inspect them
  - always try to use a prefix that defines what the object is
  - e.g., fit\_ or mod\_ for models more generally
  - or lm\_ or glm\_ to specify the function used to produce the model
  - whatever you choose, be consistent within your project

```
lm_fp <-
lm(
log(fp) ~ tense * lifetime,
subset = region == "verb" & ff > 0,
data = df_lifetime
)
```

### 1.2 Model summary

• the most straightforward way to inspect your model and the results is with summary()

```
summary(lm_fp)
```

```
Call:
lm(formula = log(fp) ~ tense * lifetime, data = df_lifetime,
    subset = region == "verb" & ff > 0)
Residuals:
    Min
              1Q
                   Median
                                3Q
                                        Max
-1.18141 -0.34282 0.00058 0.26923 1.38822
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                          0.01877 300.193 < 2e-16 ***
(Intercept)
                 5.63477
                            0.03754 0.894 0.37158
tense1
                 0.03357
lifetime1
                 -0.10130
                            0.03754 -2.698 0.00718 **
tense1:lifetime1 -0.08320
                            0.07508 -1.108 0.26828
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.4374 on 539 degrees of freedom
Multiple R-squared: 0.01712,
                               Adjusted R-squared: 0.01165
F-statistic: 3.129 on 3 and 539 DF, p-value: 0.02538
```

### 1.3 Extracting information

- lme4 objects contains lots of information
  - navigate to the model object in your Environment and explore the elements
- for example
  - nobs(model) will tell you how many data points were included in the model
  - formula (model) will give you the model formula

```
nobs(lm_fp)
```

[1] 543

#### formula(lm\_fp)

```
log(fp) ~ tense * lifetime
```

• want to check which contrasts were used in your model?

#### lm\_fp\$contrasts

# 2 Running models reproducibly

- as your models get more computationally complex, they can take time to fit
  - sometimes you also run several models in the same script, which can take a long time
- with literate programming, we want to be able to produce dynamic reports
  - we don't want to run our models anew every time we re-render our document!
  - this also might result in different results if e.g., we update a package or R version
  - or if we send the script to somebody else with a different set-up
- ideally, we would have our source code (in our .qmd script), output (results printed in e.g., HTML our PDF output), and the actual model all stored and retrievable (.rds files)

### 2.1 Storing our models

- we can save our models as R Data Serialisation files (RDS, .rds file extension)
  - saveRDS(object, filename) writes an R object as RDS file
  - readRDS(filename) reads it in

- first, you'd want to create a folder where you store your models
  - then save your models once you've run them
  - but MAKE SURE you set eval: false
  - or even better, comment out this code!

```
"" {r}
#| eval: false
saveRDS(lm_fp, here::here("models", "lm_fp"))
```

- ideally you would also set the code chunks that fit the model to echo: false
  - you don't want to re-fit the model every time you render the document
  - what's more, any differences in the re-fit model will not reflect what you're actually reporting elsewhere when using the stored RDS model

#### 2.2 Load in RDS model

• and load them in e.g., when you want to report your models in Rmarkdown

```
lm_fp <- readRDS(here::here("models", "lm_fp"))</pre>
```

# 3 Model reporting

- when you run your models, you'll want to also print the output
  - this way you avoid needing to re-run your code to see the results
- for lme4 models we can use summary()
  - as well as formula(), nobs()

#### 3.1 broom

- $\bullet\,$  the broom package also contains useful functions for reporting  ${\tt lme4}$  models
  - broom::tidy() produces the model estimates for our fixed effects

```
broom::tidy(lm_fp)
```

Table 1: Example model summary table using broom::tidy(), knitr::kable(), and kableExtra::kable\_styling()

term	estimate	std.error	statistic	p.value
(Intercept)	5.63	0.02	300.19	< .001
tense1	0.03	0.04	0.89	0.372
lifetime1	-0.10	0.04	-2.70	0.007
tense1:lifetime1	-0.08	0.08	-1.11	0.268

```
# A tibble: 4 x 5
```

```
estimate std.error statistic p.value
 term
                               <dbl>
                                         <dbl>
                                                 <dbl>
 <chr>
                     <dbl>
1 (Intercept)
                    5.63
                              0.0188
                                       300.
                                               0
2 tense1
                    0.0336
                              0.0375
                                        0.894 0.372
3 lifetime1
                                        -2.70 0.00718
                   -0.101
                              0.0375
4 tense1:lifetime1 -0.0832
                              0.0751
                                        -1.11 0.268
```

• it's also helpful to store the output as an object

```
tidy_lm_fp <- broom::tidy(lm_fp)</pre>
```

### 3.1.1 Example table

### 3.2 Publication-ready tables

- we've already seen how to produce publication-ready tables
  - TASK: produce a summary of the fixed effects of a model and feed it into our knitr and kableExtra functions to produce a publication-ready table

### 4 Mixed models

- the assumption of independence is often violated
  - due to multiple observations collected from e.g., the same person or item, language,
     etc.
- this can lead to the inflation of Type I error (the chance of a false positive)
- ullet to account for this nonindependence of data points, we can used mixed models
  - mixed because they contain fixed effects (i.e., our predictors at the population-level)
     as well as random effects (group-level overall means and predictor effects)

### 4.1 lmer()

### 4.2 Extracting model elements

• we can use the same functions as for lm() models

```
nobs(lmer_lifetime)
```

[1] 543

```
formula(lmer_lifetime)
```

```
log(fp) ~ tense * lifetime + (1 | px) + (1 | item_id)
```

• we can also check how many levels we had for each grouping factor (i.e., random effect: items and participants)

```
summary(lmer_lifetime)$ngrps["px"]
```

px

7

### summary(lmer\_lifetime)\$ngrps["item\_id"]

```
item_id
80
```

#### 4.3 summary()

```
summary(lmer_lifetime)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: log(fp) ~ tense * lifetime + (1 | px) + (1 | item_id)
  Data: df_lifetime
 Subset: region == "verb" & fp > 0
REML criterion at convergence: 582.5
Scaled residuals:
     Min
              1Q
                   Median
                                3Q
                                        Max
-2.99792 -0.67159 0.02895 0.67995 2.67790
Random effects:
 Groups Name
                     Variance Std.Dev.
 item_id (Intercept) 0.007012 0.08374
        (Intercept) 0.034124 0.18473
 Residual
                     0.154885 0.39355
Number of obs: 543, groups: item_id, 80; px, 7
Fixed effects:
                Estimate Std. Error t value
(Intercept)
                 5.63161
                           0.07245 77.736
tense1
                 0.03794
                            0.03389 1.120
lifetime1
                            0.03387 -3.044
                -0.10308
tense1:lifetime1 -0.09587
                            0.06778 - 1.414
Correlation of Fixed Effects:
           (Intr) tense1 liftm1
tense1
           -0.001
lifetime1
            0.000 0.013
tens1:lftm1 0.003 0.002 -0.002
```

#### 4.4 broom.mixed

- the broom.mixed package also contains useful functions for reporting lme4 mixed models
  - broom.mixed::tidy() produces the model estimates for fixed and random effects
  - if you only want fixed effects, use the argument effects = "fixed"
  - if you only want random effects, use the argument effects = "ran\_pars"

#### 4.4.1 tidy()

```
tidy_lmer_lifetime <- broom.mixed::tidy(lmer_lifetime)</pre>
```

```
tidy_lmer_lifetime
```

```
# A tibble: 7 x 6
 effect
          group
                                     estimate std.error statistic
                    term
  <chr>
           <chr>>
                    <chr>>
                                        <dbl>
                                                  <dbl>
                                                             <dbl>
1 fixed
          <NA>
                                       5.63
                                                 0.0724
                                                             77.7
                    (Intercept)
2 fixed
          <NA>
                    tense1
                                       0.0379
                                                 0.0339
                                                             1.12
3 fixed
          <NA>
                    lifetime1
                                      -0.103
                                                 0.0339
                                                             -3.04
4 fixed
           <NA>
                    tense1:lifetime1 -0.0959
                                                 0.0678
                                                             -1.41
5 ran pars item id sd (Intercept)
                                      0.0837
                                                NA
                                                             NA
                    sd__(Intercept)
6 ran_pars px
                                       0.185
                                                NA
                                                             NA
7 ran_pars Residual sd__Observation
                                                NA
                                       0.394
                                                             NA
```

#### 4.5 lmerTest

- is something missing from our model summary?
  - which effects were statistically significant (p < .05)?
- calculating *p*-values is not trivial for mixed models
  - lme4 doesn't do it for linear models
- we can use the lmerTest package, which also has lme4

```
tidy_lmerTest_lifetime <- broom.mixed::tidy(lmerTest_lifetime)</pre>
```

#### tidy\_lmerTest\_lifetime

```
# A tibble: 7 x 8
 effect
           group
                    term
                                    estimate std.error statistic
                                                                       df
                                                                            p.value
           <chr>
                                                                              <dbl>
  <chr>
                    <chr>
                                       <dbl>
                                                  <dbl>
                                                            <dbl>
                                                                   <dbl>
           <NA>
                                                            77.7
1 fixed
                    (Intercept)
                                      5.63
                                                 0.0724
                                                                    6.20
                                                                           1.65e-10
2 fixed
           <NA>
                    tense1
                                      0.0379
                                                 0.0339
                                                             1.12 471.
                                                                           2.63e- 1
3 fixed
           <NA>
                    lifetime1
                                     -0.103
                                                 0.0339
                                                            -3.04 468.
                                                                           2.47e- 3
                    tense1:lifeti~
                                                 0.0678
                                                            -1.41 471.
                                                                           1.58e- 1
4 fixed
           <NA>
                                     -0.0959
5 ran_pars item_id sd__(Intercep~
                                      0.0837
                                                                   NA
                                               NA
                                                            NA
                                                                          NA
6 ran_pars px
                    sd__(Intercep~
                                      0.185
                                                NA
                                                            NA
                                                                   NΑ
                                                                          NΑ
7 ran_pars Residual sd__Observati~
                                                NΑ
                                                            NA
                                                                   NA
                                                                          NA
                                      0.394
```

#### 4.5.1 Publication-ready tables

- we've already seen how to produce publication-ready tables
  - TASK: produce publication-ready tables using knitr and kableExtra for:
    - \* a table of the model estimates of the fixed effects of our model
    - \* a table of the model estimates of the random effects of our model
    - \* a table of the with both the fixed and random effects
- produce one of these tables as LaTeX code (if you haven't already)
  - can you then produce this table in Overleaf?

#### **4.5.2** Example

```
tidy_lmerTest_lifetime |>
  mutate(p.value = case_when(
    p.value < .001 ~ "< .001",
    TRUE ~ as.character(round(p.value,3))
)) |>
  knitr::kable(digits = 2) |>
  kableExtra::kable_styling()
```

Table 2: Example model summary table using broom.mixed::tidy(), knitr::kable(), and kableExtra::kable\_styling()

effect	group	term	estimate	std.error	statistic	df	p.value
fixed	NA	(Intercept)	5.63	0.07	77.74	6.20	< .001
fixed	NA	tense1	0.04	0.03	1.12	470.73	0.263
fixed	NA	lifetime1	-0.10	0.03	-3.04	468.30	0.002
fixed	NA	tense1:lifetime1	-0.10	0.07	-1.41	470.76	0.158
$ran\_pars$	$item\_id$	$\operatorname{sd}$ (Intercept)	0.08	NA	NA	NA	NA
ran_pars	px	$\operatorname{sd}$ (Intercept)	0.18	NA	NA	NA	NA
$ran\_pars$	Residual	$sd\_\_Observation$	0.39	NA	NA	NA	NA

#### 4.6 Save our model

• let's store our model locally for future use

```
```{r}
#| eval: false
lmerTest_lifetime <- saveRDS(lmerTest_lifetime, here::here("models", "lmerTest_lifetime"))
```</pre>
```

• another tip: you don't want to re-run and re-save your

# **5** Citing resources

- you should cite the packages you used to fit your models
  - as well as other package versions used (but SessionInfo() also takes care of this)
- to cite packages in-text, you can first extract the citation: with

```
citation(package = "lme4")
```

To cite lme4 in publications use:

```
Douglas Bates, Martin Maechler, Ben Bolker, Steve Walker (2015). Fitting Linear Mixed-Effects Models Using lme4. Journal of Statistical Software, 67(1), 1-48. doi:10.18637/jss.v067.i01.
```

A BibTeX entry for LaTeX users is

```
@Article{,
    title = {Fitting Linear Mixed-Effects Models Using {lme4}},
    author = {Douglas Bates and Martin M{\"a}chler and Ben Bolker and Steve Walker},
    journal = {Journal of Statistical Software},
    year = \{2015\},\
    volume = \{67\},
    number = \{1\},
    pages = \{1--48\},
    doi = {10.18637/jss.v067.i01},
  }
  • or as BibTeX citation:
print(citation(package = "lme4"), bibtex=TRUE)
To cite lme4 in publications use:
  Douglas Bates, Martin Maechler, Ben Bolker, Steve Walker (2015).
  Fitting Linear Mixed-Effects Models Using lme4. Journal of
  Statistical Software, 67(1), 1-48. doi:10.18637/jss.v067.i01.
A BibTeX entry for LaTeX users is
  @Article{,
    title = {Fitting Linear Mixed-Effects Models Using {lme4}},
    author = {Douglas Bates and Martin M{\"a}chler and Ben Bolker and Steve Walker},
    journal = {Journal of Statistical Software},
    year = \{2015\},\
    volume = \{67\},
    number = \{1\},
    pages = \{1--48\},
    doi = {10.18637/jss.v067.i01},
  }
   • then copy it to your .bib file, add a citation reference key (e.g., bates_fitting_2015),
     and cite it in-text with @bates_fitting_2015, which will be formatted as Bates, Mäch-
     ler, et al. (2015)
       - alternatively, save the package reference in Zotero and access it using rbbt
       - e.g., copy the DOI then find the Zotero button for 'Add Item by Identifier'
```

### 5.1 Citing packages

• would usually be done in Data Analysis

Linear mixed models were fit using the `lmerTest` package which produces the Satterwaite appr

Linear mixed models were fit using the lmerTest package which produces the Satterwaite approximation for degrees of freedom to calculate p-values (Kuznetsova et al., 2017).

## 6 How to report

- you would typically have a (sub)section called Data Analysis which includes all the steps taken prior to inspecting your results
  - i.e., how you prepared and analysed your data
  - this should be information stated in a pre-registration but in the past tense
- this would typically be followed by a Results section
  - containing the results from these cleaning/analysis steps

### 6.1 Data analysis

- should include, e.g.,
  - model summaries in tables and/or in-text
  - any inclusion/exclusion criteria for participants or observations
  - any transformations performed on dependent or independent variables
    - $\ast$  e.g., we log-transformed first-pass reading times, and used sum contrast coding for our two 2-level predictors
    - \* we should therefore define which levels were coded as what value (+/-0.5)
- model selection
  - what model did you start with? E.g., maximal model given the data and research questions (à la Barr et al., 2013)
  - how did you handle convergence issues? E.g., parsimonious model selection (à la Bates, Kliegl, et al., 2015)

#### 6.2 Results

- this would be followed by a section called Results, which reports e.g.,
  - the number of participants and trials ex-/included in analyses
  - possibly raw means and 95% confidence intervals
  - model formula
  - information about the number of observations per model
- a structure overview of model results

### 6.3 Example model report

- in an Rmarkdown script where you write up a paper or thesis, you could include the following hidden code chunk
  - i.e., set {r, echo = FALSE}, which is slightly different than the Quarto #| eval: false chunk option syntax

```
pacman::p_load(lme4, lmerTest, tidyverse)
lmerTest_lifetime <- readRDS(here::here("models", "lmerTest_lifetime"))</pre>
```

```
A linear mixed model was fit to log-transformed first-pass reading times, with `r summary(lmanner) An effect of *lifetime* was found in first-pass reading times at the verb region (Est = `r s
```

### 6.4 Output

A linear mixed model was fit to log-transformed first-pass reading times, with 7 participants and 80 items. The final model formula was log(fp) ~ tense \* lifetime + (1 | px) + (1 | item\_id).

An effect of *lifetime* was found in first-pass reading times at the verb region (Est = -0.1, t = -3, p = 0.0025). A significant effect of *tense* was not found found (Est = 0.038, t = 1.1, p = 0.26), nor an interaction of *lifetime* and *tense* (Est = -0.096, t = -1.4, p = 0.16).

# Learning objectives

Today we...

- run our first linear (mixed) model
- produce model summary tables
- report analyses and cite packages
- report model results reproducibly

#### Session Info

```
print(sessionInfo(), locale = F)
R version 4.4.0 (2024-04-24)
Platform: aarch64-apple-darwin20
Running under: macOS Ventura 13.2.1
Matrix products: default
BLAS:
        /Library/Frameworks/R.framework/Versions/4.4-arm64/Resources/lib/libRblas.0.dylib
LAPACK: /Library/Frameworks/R.framework/Versions/4.4-arm64/Resources/lib/libRlapack.dylib;
attached base packages:
[1] stats
              graphics grDevices datasets utils
                                                       methods
                                                                  base
other attached packages:
 [1] lmerTest_3.1-3
                          brms_2.21.5
                                              Rcpp_1.0.12
 [4] broom.mixed_0.2.9.5 broom_1.0.6
                                              lme4_1.1-35.3
 [7] Matrix_1.7-0
                          lubridate_1.9.3
                                              forcats_1.0.0
[10] stringr_1.5.1
                          dplyr_1.1.4
                                              purrr_1.0.2
[13] readr_2.1.5
                          tidyr_1.3.1
                                              tibble_3.2.1
                          tidyverse_2.0.0
[16] ggplot2_3.5.1
loaded via a namespace (and not attached):
 [1] tidyselect_1.2.1
                           viridisLite_0.4.2
                                                100_2.7.0
 [4] fastmap_1.2.0
                           tensorA_0.36.2.1
                                                pacman_0.5.1
 [7] digest_0.6.35
                           timechange_0.3.0
                                                estimability_1.5.1
[10] lifecycle_1.0.4
                          magrittr_2.0.3
                                                posterior_1.5.0
[13] compiler_4.4.0
                           rlang_1.1.4
                                                tools_4.4.0
[16] utf8_1.2.4
                           yaml_2.3.8
                                                knitr_1.47
[19] bridgesampling_1.1-2 bit_4.0.5
                                                here_1.0.1
```

[22]	xml2_1.3.6	abind_1.4-5	numDeriv_2016.8-1.1
[25]	withr_3.0.0	grid_4.4.0	fansi_1.0.6
[28]	xtable_1.8-4	colorspace_2.1-0	future_1.33.2
[31]	globals_0.16.3	emmeans_1.10.2	scales_1.3.0
[34]	MASS_7.3-60.2	cli_3.6.2	mvtnorm_1.2-5
[37]	rmarkdown_2.27	crayon_1.5.2	generics_0.1.3
[40]	RcppParallel_5.1.7	rstudioapi_0.16.0	tzdb_0.4.0
[43]	minqa_1.2.7	splines_4.4.0	bayesplot_1.11.1
[46]	parallel_4.4.0	matrixStats_1.3.0	vctrs_0.6.5
[49]	boot_1.3-30	jsonlite_1.8.8	hms_1.1.3
[52]	bit64_4.0.5	listenv_0.9.1	systemfonts_1.1.0
[55]	glue_1.7.0	parallelly_1.37.1	nloptr_2.0.3
[58]	codetools_0.2-20	${\tt distributional\_0.4.0}$	stringi_1.8.4
[61]	gtable_0.3.5	munsell_0.5.1	furrr_0.3.1
[64]	pillar_1.9.0	htmltools_0.5.8.1	Brobdingnag_1.2-9
[67]	R6_2.5.1	rprojroot_2.0.4	kableExtra_1.4.0
[70]	vroom_1.6.5	evaluate_0.23	lattice_0.22-6
[73]	backports_1.5.0	renv_1.0.7	rstantools_2.4.0
[76]	svglite_2.1.3	coda_0.19-4.1	nlme_3.1-165
[79]	checkmate_2.3.1	xfun_0.44	pkgconfig_2.0.3

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